Abstract: Shared life in Go – generalized hanezeki

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Abstract: The game of go [weiqi (Chinese/C)/ baduk (Korean/K)] has a number of distinct types of shared life – better known seki (Japanese/J) [shuang1 huo2 (C)/ bik (K)]. In some positions of shared life it is possible to capture. Hanezeki is a position with the possibility of both a simple capture, and a separate mutual capture which allows immediate recapture for one side. We provide a complete classification of hanezeki, describe how to create all 154 different types, and give some examples.

Some restrictions
We will use a basic Chinese method of counting – this will let us add stones inside a territory without changing the score. Furthermore, we will normally deal with "terminal" positions – this is effectively a position at the end of the game, but with the further restriction that if either player plays a move they will lose more points than they gain. A terminal seki is one where it is unsafe/unwise for either player to play another move

Capture -- immediate re-capture (CIR)
   Hanezeki (jeochim bik (K)) has been known for at least 300 years, and maybe for more than 650 years. Fig 1 appears in some, maybe only later, editions of *XuanXuan QiJing* (Gengen Gokyo(J)/ Hyun Hyun Ki Kyung (K)) – by Yan TianZhang & Yan DeFu, and first published in 1347(or 1349). Fig 1 also appears as item 550 in *Guan Zi Pu* (Kanzufu(J)/ Gwan Ja Bo(K)) by Guo BuiLing in 1689, and is mentioned in a recent article [Haruyama1987].
In 1713 Inoue Dosetsu Inseki published *Igo Hatsuyoron* (also known as *Fudanzakura*(J)/ FaYangLun(C)/ Bal Yang Hyun. Gam Non (K)) in 1713. It contained a hanezeki. This problem seems to have been lost, and rediscovered in approximately 1982 by Fujisawa Shuko, and then published by him in [Fujisawa1993], and discussed in [GW1982] and [Feldmann2005]. It is a beautiful full board problem, using combination "C"-"b" in Table 1). Another previously published example of hanezeki is shown in Fig 2 [Shimada 1958]. In both of these examples, if either player captures any stones, then all their stones will eventually be killed.
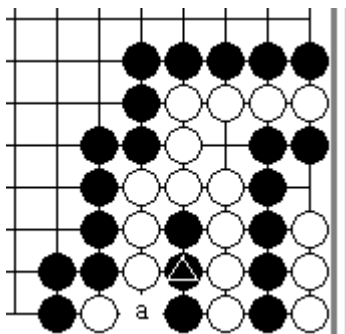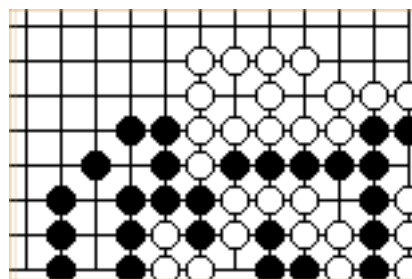

Fig 1: XuanXuan QiJing hanezeki

Fig 2: Shimada's hanezeki

These consist of two components – a capture-immediate-recapture (CIR) component and a big-eye/nakade (chijungsu(K)/ zhong1 shou3(C)) component. The nakade components are well known. Fig 3 shows all the basic CIR components based on these centre/edge nakade captures – however we do not consider here those CIR which are snapback (uttegaeshi(J)/ hwan-gyeok (K)/ dao4 tie1(C)). Fig 3 also includes the degenerate -- 1-, and 2-stone – CIRs (X, Y, and Z). Note that, the pyramid-four capture gives us four different CIR configurations – Fig 3 H, I, J, and K. They may behave differently if Black captures first – see Table 1. Similar considerations apply for other captures. In Fig 3, in each case the single white stones can be replaced by two, or more, stones – see Fig 4. The effects of this are discussed later – see Table 1.
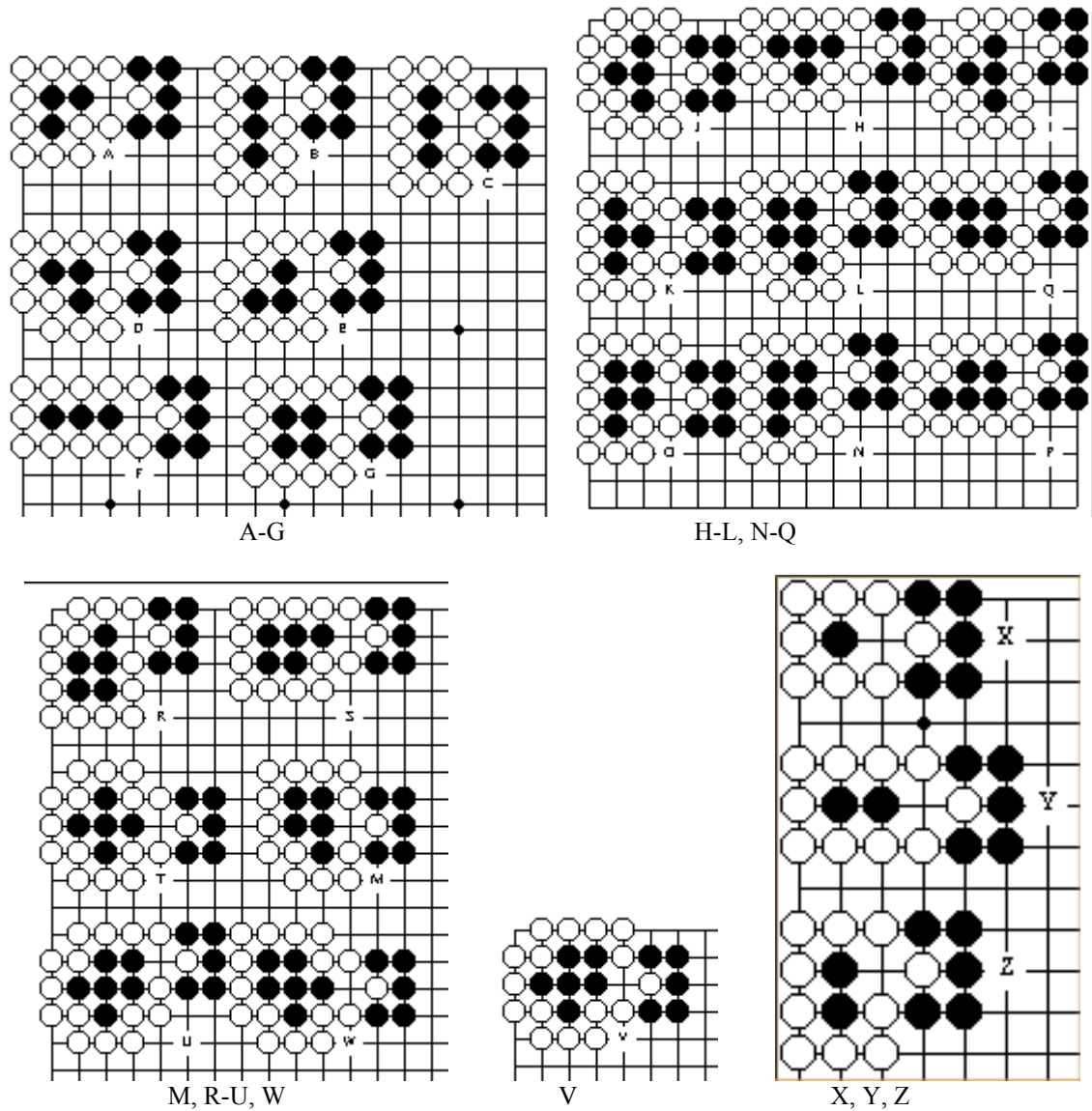
A-G   H-L, N-Q

M, R-U, W   V   X, Y, Z

Fig 3: Centre/edge CIRs

In Fig 1 and Fig 2, a key condition is that Black's capture of three stones in a nakade at the right, and White's capture, result in the same number of liberties. If these numbers are different then the position cannot be a seki. Figs 3A-W show the 23 different configurations of 3-6 (black) stones (in the middle of the board) such that, *both* their capture leaves a possible nakade, *and* Black can capture a single white stone, but if he does so his capturing chain can itself be captured. Of these 23 positions, 15 can also exist on the edge of the board – simply remove the top row of stones in each sub-figure. The remaining 8 that cannot exist at the edge are shown inside [square brackets] (e.g. "[C]") in Table 1. This gives a total of 38 distinct capturable configurations of (black) stones.
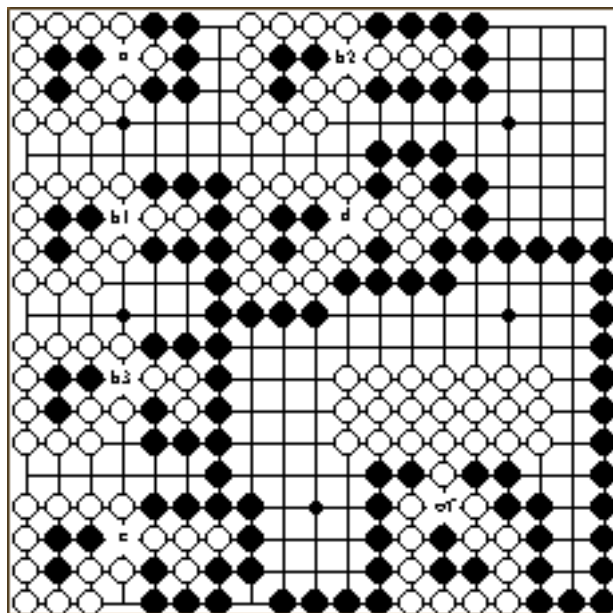
Fig 4 a-f: CIR – shapes/sizes of initial capture

In Fig 2, the black play at "a" captures a single stone – the corresponding capture in Fig 1 captures three white stones. Fig 4 a-f shows the 6 possible, (interestingly) different captures of white stones. "b1", "b2", and "b3" look different, but behave in the same way as each other (except for number of stones captured). "e" and "f" are quite similar – their differences are discussed later. "a", "b", and "c" can all occur both at the edge of the board, and in the centre – "d", "e", and "f" cannot occur at the edge. All combinations of "a"-"f" with the 38 capturable configurations of Fig 3, are given in Table 1, showing the result of initial capture by Black.

What happens in a fight between one of these 6 x 38 combinations, and nakade of the same size? In all cases, if White plays first then White loses. Table 1 shows what happens when Black plays first in each of the combinations. All entries with "W" are simple hanezeki; all with "B" mean that Black can capture at any time – i.e. not seki; all others – e.g. "3/3" -- mean that White can win but only if they win a ko. Fig 1 appears in Table 1 as "D"-"b", and Fig 2 appears as "B"-"a".

Table 1: Hane seki *versus* nakade – Black plays first -- who wins?

		Capture of white stones at the right (Figs 3 & 4)					
		1	*2 = 3*	*4*	*5*	*d+1 lib*	*> d+1 lib*
		a	b	c	[d]	[e]	[f]
(1)	X	B+4	B+3	B+2	[B+1]	[B]	[W]
(2)	Y	W	W	W+k	[W+k]	[W+1]	[W+1]
	Z	W	W	W+k	[W+k]	[W+1]	[W+1]
(3)	A	W	W+1	W+1	[W+1]	[W+1]	[W+1]
	B	W	W+1	W+1	[W+1]	[W+1]	[W+1]
	[C]	[W]	[W+1]	[W+1]	[W+1]	[W+1]	[W+1]
	D	W	W+1	W+1	[W+1]	[W+1]	[W+1]
	E	W	W+1	W+1	[W+1]	[W+1]	[W+1]
	F	W	W+1	W+1	[W+1]	[W+1]	[W+1]
(4)	G	B	W	W	[W+1]	[W+2]	[W+…]
	[J]	[B]	[W]	[W]	[W+1]	[W+2]	[W+…]
	H	3/3 *	2/2	W	[W+1]	[W+2]	[W+…]
	I	3/3 *	2/2	W	[W+1]	[W+2]	[W+…]
	[K]	[3/3 *]	[2/2]	[W]	[W+1]	[W+2]	[W+…]

(5)	L	4/4	3/3	2/2 *	[W]	[W+1]	[W+…]
	L'	*5/5 **	*4/4 **	*3/3 **	---	---	---
	Q	4/4	3/3	2/2 *	[W]	[W+1]	[W+…]
	Q'	*6/5 **	*4/4 **	*3/3 **	---	---	---
	[O]	[4/4]	[3/3]	[2/2 *]	[W]	[W+1]	[W+…]
	N	B+1	6/5	5/4	[W]	[W+1]	[W+…]
	P	B+1	6/5	5/4	[W]	[W+1]	[W+…]
	R	B+2	4/3	4/3	[W]	[W+1]	[W+…]
	S	B+2	4/3	4/3	[W]	[W+1]	[W+…]
	[T]	[5/4]	[4/3]	[4/3]	[W]	[W+1]	[W+…]
	[M]	[B]	[B]	[B]	[B]	[W]	[W+1…]
(6)	U	8/5	7/4	5/2	[0/3&1]	[W]	[W+1…]
	U'	*10/9*	*9/8*	*9/8*	---	---	---
	[V]	[7/5]	[6/4]	[5/3]	[0/2&1]	[W]	[W+1…]
	[W]	[10/5]	[9/4]	[9/4]	[0/1&1]	[W]	[W+1…]

Notes for Table 1:

- [ ] = These shapes exist in the centre, but not on the edge of the board. All others (except L', Q' and U') exist both in the centre, and on the edge.
- (L, *L'*), (Q,*Q'*), & (U, *U'*): e.g. - *L'* shows the different outcome when L is on the edge.
- * = If the white nakade can go via the 2x2 block 4, White can play this as a ko threat – then White has to find one less ko threat – e.g. 3/3 will become 2/3
- B = Black wins; B+2 = wins with 2 extra liberties; similarly for W, W+1, W+2, etc.
- **w/b** = White must play at least **w** (answered) ko threats, and White must ignore at least **b** Black ko threats to win the fight. **0/2&1** is unusual – White need play no ko threats, may respond to the first 2 Black ko threats, and then ignore the next one.

Conclusions

There are 128 simple centre/edge hanezeki, and 59 which require White to fight a ko. Black will not start these if he will lose the ko fight – these ko fights require non-removable ko-threats.

We later extend the analysis to produce variants created from four further simple corner-specific captures. We do the same for the three complex corner nakade – bent four, rectangular six, and Murashima's 8-stone capture (giving 10,000 year ko – see Fig 5). In doing this we create an additional 26 simple hanezeki, and another 28 which depend on ko. Altogether, we have now catalogued all 154 simple hanezeki, and a further 87 with ko.

We also consider configurations where two CIRs oppose each other – ko is important.

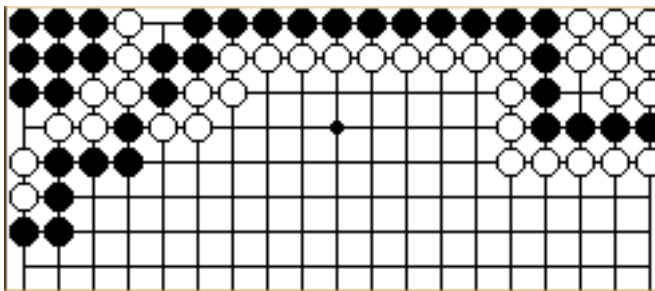Fig 5 and Fig 6 are two extreme examples of hanezeki.
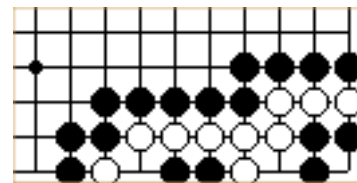

Fig 5: Murashima's ko hanezeki

Fig 6: Smallest hanezeki