Shared life in Go – an overview

Harry Fearnley, 38 Henley Street, Oxford, OX4 1ES, UK.   email: harry@ZZgoban.demon.co.uk

Abstract
The game of Go [weiqi (Chinese/C)/ baduk (Korean/K)] has a number of distinct types of shared life [seki (Japanese/J)/ shuang1 huo2 (C)/ bik (K)]. There are more kinds of shared life than there are of independent, unconditional, life. We provide an overview, and outline an approach towards a full classification of shared life, while referring to the relatively few known publications. We include positions in which capture is possible, but not desirable – e.g. hanezeki, and generalized nakade. We describe the components that are available, and indicate how we can systematically construct seki configurations using these components, and give some examples. We indicate what work is required to catalogue fully all seki configurations.

Keywords: go, baduk, weiqi, shared life, seki, shuang huo, bik, ko, jie, pae, non-removable ko threats, centre play, nakade, dianyan, chijung, hanezeki, jeochim bik, snapback, uttegaeshi, hwangyeok, daotei, daopu.

Introduction
When playing Go, it is very desirable to have a full catalogue of target positions. These target positions are shapes of various kinds – at one end of the spectrum we might have guaranteed 2 eyes, or life with seki; at the other end a few liberties, or a simple ko. This catalogue would include (an abstract representation of) all those positions where a group is unconditionally dead, all *minimum* configurations of independent life, and all possible configurations of shared life, as well as various conditional/intermediate positions. When we are playing, such a catalogue provides a list of subgoals – target configurations we want to avoid, or to create. Here we outline some work done to create a catalogue of shared life (seki), and indicate some further work to be done.

While such targets are helpful to human players, they are arguably much more helpful to computer programs, helping them dramatically to prune the game tree. When a computer program plays it constructs a game tree -- an abstract representation of all the possible limited sequences of moves at a particular point in the game. It then removes (prunes) bits with inferior outcomes. In computer Go, position evaluation functions are notoriously weak ([Bouzy2001]), and full catalogues could provide some help in increasing effective search depths. For computer programs such catalogues offer another potential use – as a database of positions to test both their algorithms, and their "reasoning" ability.

Catalogues/databases can be valuable -- in (Western) Chess much work has been done to catalogue fully some endgame positions. Notable is the work of Donald MICHIE, Ivan BRATKO, and others at Edinburgh – e.g. see [Bratko1980]. With the help of a computer, they created a database of all the distinct arrangements of (King and Rook) vs (King and Knight) – KRKN. They found that the longest possible forced sequence, leading to checkmate, but without a capture, was 52 moves long. This showed that the existing rules of Chess were inadequate. Furthermore, working with strong Chess players, they showed that no published advice on playing the KRKN endgame was either complete, or completely correct. They were also able to replace the best available book with brief, and simple, heuristics, which were provably correct. We know of no comparable work in Go.

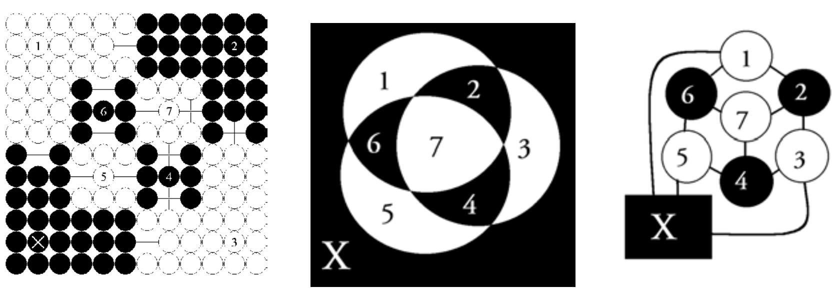
There is some sporadic work to increase our knowledge of dead shapes – e.g. [Pauli2004a] has increased the classic [Dosetsu1713] 16-stone nakade capture to 17 stones – but still no catalogue of all the dead shapes. Such a catalogue will be needed to complete the work on "Capture, Delayed Re-capture" (CDR) – see later.

Stones that are part of a group which has two eyes are independently alive. At the end of the game, all other live stones will be part of a shared life configuration.

We make some simplifying assumptions in order to make our work easier. We use a Chinese method of counting, and we normally deal only with "terminal" positions, where it is unsafe/unwise for either player to play another move – whoever plays first will lose more points than they gain. This should not be confused with a slightly weaker condition which we will meet later -- one in which it is unsafe/unwise for one player to play, but safe, and possibly profitable, for the other player to play – the other player's move may be a non-removable, one-sided, ko-threat. These positions may be thought of as "unstable" sekis, and will usually stay unplayed until the end of the game. The two assumptions – Chinese rules, and terminal

positions – can later be removed, and this will probably affect our results only predictably, and to a small extent.
We make one other important simplifying assumption: there are no “loopy”/cyclic positions (ko (dianyan(C)/ pae(K)), eternal life (chosei(J)/ changsheng(C)/ jangsaeng(K)), etc) already on the board. In some sekis without an *initial* cyclic position it will be possible for one player to choose to start a sequence which eventually gives a cyclic position (usually a ko). These will be considered, although the positions are not terminal, and they sometimes depend on the existence of non-removable ko threats – various examples are already known, including “bent-four-in-the corner”, and a position due to FUJIMURA Yoshikatsu, described in [Grant1984]. If we later allow initial cyclic positions, we may have to make a number of adjustments to some of our analyses.
We define a “chain” as a collection of stones of the same colour that are already *fully connected* to each other. A “group” may consist of one, or more, “chains”. “Group” is a less precise term – the meaning is always clear when describing terminal two-eyed groups. However, in shared life, especially where mutual capture is possible, it is sometimes not obvious which chains belong to a group – it can depend on the flow of play. We may therefore sometimes prefer to discuss chains, instead of groups.

Configurations for independent life
Independent life offers fewer complications than shared life. Despite this, there is not yet a definitive classification of all possible configurations of simple 2-eyed groups. [Fearnley2003] catalogues the topologically different configurations of from one to six two-eyed groups – two configurations are the same if their structure does not change when the colours are exchanged. Similarly, the exact position, or size, of the groups, is not important. All that is important is which groups touch which others, and which groups touch the edge of the board. There are (at least) 1, 2, 4, 14, 42, and 168 (total 231) such topologically different configurations which have 1 to 6 groups, respectively. Many configurations with substantially more than six groups are known – this includes the configuration with the most (31) 2-eyed groups that can be fitted onto a standard 19x19 go board, and is due to Ger HUNGERINK ([Fearnley2001]). It will be difficult to provide a full analysis for a larger number of groups. The numbers of configurations are substantially smaller if we are concerned only with which groups touch each other, and we do not care about how they do so, or about their access to the edge of the board. In this case, as well as an abstract “map”, we can use either of two equivalent representations – a graph, or an adjacency matrix.



		Black			
		4	6	2	X
White	1	0	1	1	1
	3	1	0	1	1
	5	1	1	0	1
	7	1	1	1	0

Fig 1: Octahedral seki: board, map, graph, and adjacency matrix

In Fig 1 – the groups are numbered to help us compare each representation with the other representations. Next to the board position, we see the corresponding map, and then the graph which consists of a black/white blob (vertex) for each group, and a line (edge) connecting each pair of groups that have adjacent stones. The corresponding adjacency matrix has a row for each white group, and a column for each black group – there is a “1” in each cell where the groups touch (are adjacent), and a “0”, elsewhere. A similar idea for graphs and matrices, is used in [Gurvich1981] in discussing sekis, except that their paper has a separate edge for each shared liberty, and the number in each cell of the matrix is therefore the number of connecting edges (shared liberties). Such abstract representations lose some information about connectedness, where a pair of groups is connected by multiple distinct routes, but they keep the essential information about connections, and disconnections. The abstractions make it easier to reason about complicated configurations, particularly to decide if two positions are equivalent with regard to questions of life and death. However, they are probably much better suited to computers than humans, since sometimes it is not obvious what the abstract representation of a particular board position is. Furthermore,

some work may be required to convert a graph, or matrix, to the required canonical form, so that we can recognise it.
In independent life, other than topology, the only other factor that can be changed is the types of the eyes. These differences of eye type do not increase the number of possible, topologically different, configurations. For example, in configurations containing false eyes – such as the two-headed dragon – we can replace the false eyes with ordinary eyes without having to change the topological relationships. The false eyes of a two-headed dragon require us to have at least one group of the opposing colour inside it – obviously, the converse is not true.

Configurations for shared life

Positions supporting shared life are more complicated than those which have independent life only. There are also many more of them. Most, probably all, of the "independent life" configurations will have a corresponding, topologically similar, seki. Because these sekis do not require two eyes, they will usually be smaller. Sometimes, it may be hard to find examples of such comparable seki configurations – they may involve multiple shared liberties, eyes of various sizes, uneven numbers of shared liberties, etc.
A particular shared life configuration involves two or more groups. When no capture is possible, all members of the same seki configuration share liberties either directly with the other groups, or (recursively) with groups which, in turn, share liberties with the other groups. When capture is possible, the same is also normally true. However, an obvious exception is a capture such as Fig 20, if it were part of a seki. Two other exceptions (relating to hanezeki) can be seen in Fig 31a, and Fig 31b. These are related to Fig 28, and each contains two linked sekis – Black will only capture in the seki at right if the loss there is balanced by the gain on the left.
Under our assumptions, in *independent* life all empty points (liberties) are adjacent to only one group -- they are the one-point-eyes. However, in seki, there are always some liberties which are adjacent to chains of both colours. In terminal positions, all *shared* liberties, are necessarily part of a shared life configuration.
Sometimes, there are also liberties that are adjacent to two different chains of the *same* colour – see Fig 2. Exceptionally, there may be a liberty which is adjacent to two white chains, and simultaneously adjacent to two black chains (see Fig 3, and Fig 7), or even simultaneously to three black, and one white chain (see Fig 4) – yet, neither player wants to play the connection/disconnection. Such configurations are not fully connected, and make possible more complex topological arrangements than with two-eyed, independent life alone. There is apparently no published catalogue of such positions, although there seem to be only a few types. Note that Fig 5 is related to Fig 2, but is different. Fig 2 is a normal seki, whereas Fig 5 is not – in Fig 5, if Black plays any move then Black will lose all their stones. However, if White plays then Black has to respond – White has got a one-sided, non-removable, ko threat – moreover, it is a threat which loses no points. Only a small proportion of sekis have this possibility.
In seki, there seems to be another source of potential complication – non-symmetric liberties. Simple non-symmetric liberties do not involve ko -- see Fig 6a, where Black has a guaranteed extra liberty, and Fig 6b, which shares the further property that when Black plays a shared liberty, White will (probably) want to answer by playing the other one. It seems unlikely that these simple non-symmetric liberties can play a role in *terminal* seki, particularly where no capture, or only immediate capture, is possible. However, if combined with some other chains having two, or more liberties, we are less sure. Complex, non-symmetric, liberties (which include the possibility of ko) are altogether more promising. Two common complex non-symmetric liberties are shown in Fig 6c, and 6d. Another is shown in the 1989 Japanese Rules of Go – see [Hansen1989], their Example 13. A closely related example is shown in their Example 23, where the whole configuration can be converted to a simpler seki. We can also view more complicated positions, such as Fujimura's position [Grant1984], as being extreme examples of non-symmetric liberties.

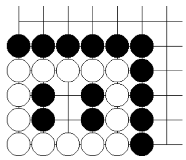


Fig 2: 2-1 shared

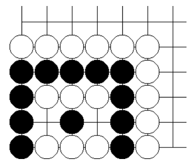


Fig 3: 2-2 shared

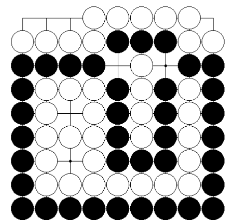


Fig 4: 3-1 shared

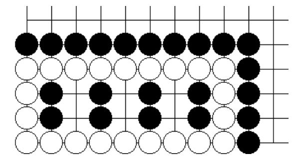


Fig 5: Non-removable threats

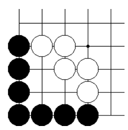


Fig 6a

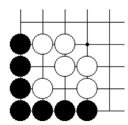


Fig 6b

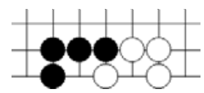


Fig 6c: complex

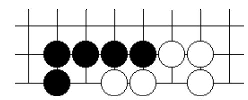


Fig 6d: complex

Fig 6: Non-symmetric "shared" liberties

No capture

In some sekis no capture is possible, and each group/chain has at most a single one-point eye. [Gurvich1981] discuss fully connected seki -- i.e. no empty point is adjacent to two, or more, groups/chains of the same colour – which share identical topological limitations with arrangements of two-eyed groups.. This excludes sekis such as the one in Fig 4. Their analysis uses both graphs, and "seki matrices" – see our discussion above. Each graph corresponds to a seki matrix, and vice-versa. This approach is particularly appropriate for seki without capture, and where no two chains/groups of the same colour may share liberties. In their analysis they extend the seki matrix so that they can include groups with a one-point eye. They also include sekis in which the numbers of stones in each group are finely balanced, and it is the relative sizes of the groups which means that both players avoid playing. Balancing of group sizes is less critical in hanezeki, and is not explicitly analysed in [Fearnley2005a], but is clearly of interest in configurations such as those in Figs 31a, 31b, 34, and 36, also potentially in all hanezeki (see Fig 30).

When capture is not possible we have to consider only no-eyed groups/chains, and one eyed groups with single point eyes, possibly combined with each other. If all groups are of one type – either all no-eyed, or all one-eyed – it is possible to create sekis involving more chains, and constructing them is simply a topological problem, in which we also have to balance liberties.

There is no published catalogue of all such configurations. Ger HUNGERINK has published -- see [Fearnley2001] -- a seki thought to have the most (129) distinct chains – see Fig 7. Note that this is very many more than the largest number (31) of two-eyed groups that can be fitted on a 19x19 board.

There has been some recent discussion on the "rec.games.go" usenet newsgroup, about various contrived configurations, especially regular/symmetric ones. Because of their having four areas/groups meeting at a point, the prime candidates are the octahedron, and the chequerboard. In 2004 Bill TAYLOR initiated a discussion thread [RGG] including both the chequerboard, and a regular eight-group configuration based on an octahedron – a variant is illustrated in Fig 1, above. [Pauli2004] visualizes this as the octahedron's dual (a cube). Such regular positions make it easier to fit more groups in a smaller space, but are not so helpful when creating configurations with differing numbers of shared liberties. We can see something of this if we consider a large regular chequerboard configuration of groups – if a black group in the middle is killed/removed, we see that the four adjacent white groups are merged, and this has a substantial effect on the overall structure.

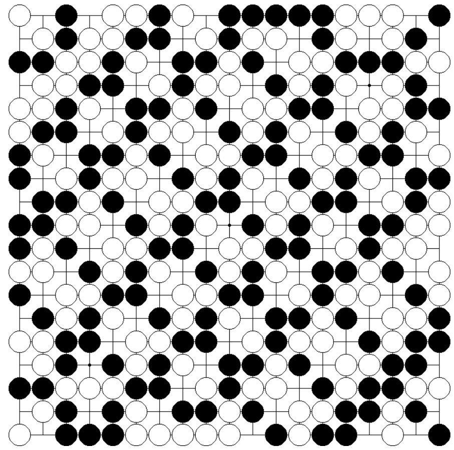


Fig 7: Hungerink's 129-chain seki

Capture

In seki we sometimes find stones which can be captured – perhaps on the next move. They are left on the board not captured, because to capture them would lead to a worse result than simply leaving them alone. In all cases, the capture does not guarantee independent life, but leads to various other possibilities, such as: a second eye (but only after winning a ko); one guaranteed eye, together with some liberties (perhaps with ko); or simply some liberties (perhaps with ko). Sometimes, there will be a choice between, on one hand, obtaining some liberties, and, on the other hand, obtaining fewer liberties, but with an additional ko. This choice may be the attacker's, and/or the defender's. Examples of all of these possibilities can be found below, and in discussions of generalized hanezeki [Fearnley2005a], and seki with generalized nakade [Fearnley2005b].

Another factor that may affect outcomes is whether an initial capture can be made immediately, or only after some further plays – delayed capture. Delayed-capture positions may go via immediate-capture positions – it will be necessary to make use of this in extending the analyses reported here. An example of delayed capture might be one of the precursor positions for bent-4-in-the-corner, or a predecessor of a simple nakade capture. Positions involving delayed capture have not been fully catalogued yet, and can be expected to provide a fruitful source of unusual seki configurations – however they will not be considered further here. We expect that "delayed capture" configurations will require more sophisticated analysis, and that most, but not all, of them will be precursors of the immediate-capture configurations discussed here.

We will construct shared life configurations systematically, using combinations of standard components. As with all considerations of life and death in Go, we need to evaluate the number (and size) of eyes, and the number of liberties, as well as their type (normal/ ko).

Simple capture – nakade

The best-known example showing possible capture involves ordinary "nakade" (dianyan(C), chijung(K)) captures – a lump of from three to six stones is captured, the capturer is left with no cutting points, but does not have a guarantee of independent life. Some of the standard 3-6 stone captures give fewer liberties when captured in the corner. Furthermore, there are six cases where the corner (1-1 point) is empty, and is surrounded by a white "nakade" group, which can be captured immediately by Black. In seki, these positions differ from normal nakade – Black will require at least two "external" liberties. In addition to these shapes, there are also three other shapes in the corner which may lead to ko – these complex corner nakade are the classic bent-four, rectangular six, and an 8-stone capture (Murashima's ko – see both corners in Fig 11) leading to 10,000 year ko (mannen ko(J)/ wan nian jie(C)/ mannyeon pae(K)). Two other candidates for use in the corner – *step four*, and *butterfly seven* – may require special consideration in Japanese-style rules, but cannot be used by us in constructing seki.

A paper on generalized nakade capture in seki [Fearnley2005b] examines all combinations of one, or two, black groups in contact with all of two, or more, white groups, and where generalized nakade captures of all kinds are possible – this includes simple classic nakade captures (in the centre/edge and the corner), as well as the complex ones in the corner, which eventually involve ko. The various captures lead to the capturer having variously 0, 1, 2, 3, 4, 4 (or 1 + ko), 6, 6 (or 1 + ko), 7, 9 (or 3 + ko), and 11 liberties.

These building blocks can be used to construct seki of various kinds. There are several other types of configuration in which nakade captures might also be used.
The simplest type of seki with nakade capture has a single group of each colour, and capturing identical-valued shapes. After this, the simplest such seki have a single (black) group involved with several identical (white) groups, and sharing only one liberty with each of them – each white group can make an identical capture. These include "unstable" sekis – Black cannot play safely; however White can start a sequence of captures, such that the position finishes as seki, but White has made more profit than Black has done – see Fig 8.
[Fearnley2005b] provides a detailed analysis, and shows several configurations not catalogued before. The analysis relies on the observation that if a configuration is really one of shared life, then we know that a *minimum* requirement is that, for each group, the *total* number of (both internal/eye and external) liberties must be equal to the total number for all groups in the same seki. A formula us derived, and is used to create a list of candidate configurations – we can then reject the few where there is no defence. For example, where a single black group is involved with several white ones, when White starts the attack by capturing (playing an internal liberty), Black's defence is to play on as many of these internal liberties as possible. If the black group, and this white group, now have nakade of the same size, and Black has enough shared liberties, then Black can create a temporary stalemate between the two groups – Black can now attack, and eventually kill, all the white groups. If Black cannot do this then the black group dies.
With the limiting conditions, there are the following sekis involving two white groups:

- Fig 8: unstable seki – White plays at any time to get a terminal seki, and gets 14 points;
- Fig 9: possible unstable seki – if either (or both) of the black lumps is the hat-4 shape, then White captures eight black stones, and obtains another seki, gaining 11 points -- see also [Feldmann2005]; If both black lumps are block-4, then White cannot play – it is a terminal seki.
- Fig 10: unstable seki – White can gain 17 points at any time; Black cannot play safely.
- Fig 11: White choice – seki, or ko to kill Black
- Fig 12: Black choice – seki, or (favourable) ko to kill White

There is one configuration involving three white groups – see Fig 13. See also [Feldmann2005].

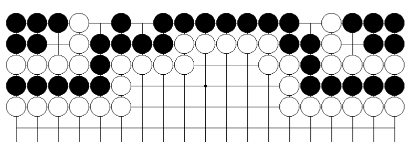


Fig 8: Equal 1-3 -- unstable

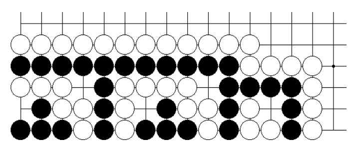


Fig 9: Equal 2-4 -- Possible unstable seki

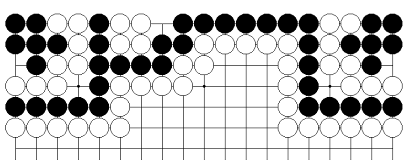


Fig 10: Equal 4-6 -- unstable

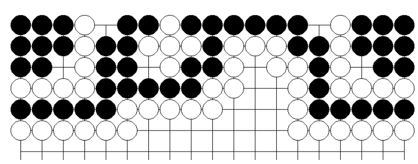


Fig 11: Equal 7-9 – seki/ Black dies (ko)

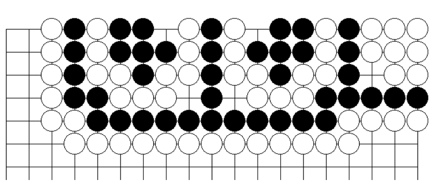


Fig 12: Equal 9-11 – seki/ Black dies (ko)

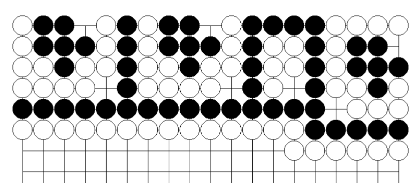


Fig 13: Largest centre/edge nakade combination

There is another type of position that has some similarities with seki – one player can capture many stones, but both players would prefer the other player to play first (as in seki). A well-known example is Fig 14a -- if White plays first then all white stones die, whereas if Black plays first then one white group lives – Black decides which one lives. [Fearnley2005b] finds that four other centre/edge configurations exhibit similar behaviour. The most extreme example is shown in Fig 14b, where all five white groups die if White plays first, but only four die if Black plays first.

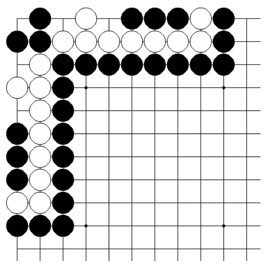


Fig 14a: One-or-two die

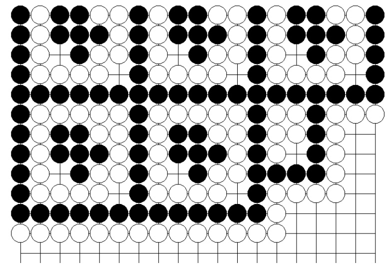


Fig 14b: Four-or-five die

Different sized nakade captures; different shared liberties

[Fearnley2005b] uses the same approach as in the previous section to find three families of seki, where White's only possible attack is first to capture; Black's defence is to take those internal liberties, and to threaten a temporary seki:

- Fig 15 – terminal seki – see also [Feldmann2005]
- Fig 16 -- terminal seki
- Fig 17 -- White is safe, but can choose to kill Black with ko.

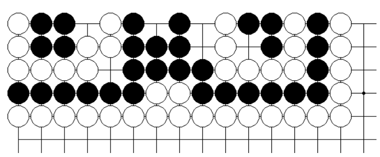


Fig 15: Unequal 1-2-4 -- terminal

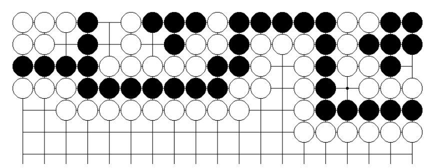


Fig 16: Unequal 3-4-6 -- terminal

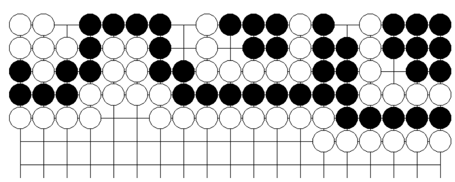


Fig 17: Unequal 6-7-9 – seki/ Black dies (ko)

In the three families above, the only ones which are terminal seki are those shown in Figs 15-17. In each of these figures, we can add an equal number of liberties to each set of shared liberties to give a similar, but non-terminal, seki where Black has one-sided, non-removable, ko threats.

[Fearnley2005b] further extends all the previous analyses from one black group to two, or more, black groups – for example, see Fig 18, below.

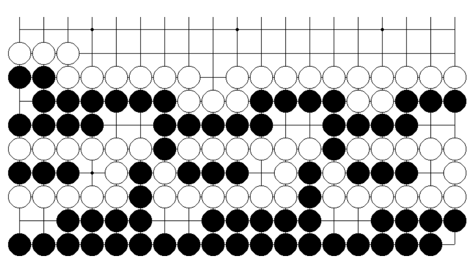


Fig 18: 2 black *versus* 3 white : 1-3

The complex corner nakade involve ko – these remain to be fully investigated. More work also remains to be done, both including more complicated topologies, and with more groups.
The nakade captures are one-sided, captures – if Black can capture some white stones, those same white stones cannot capture any black stones. In addition to these one-sided nakade, there are also several other situations where two-sided, mutual, capture is possible. Sometimes, immediate recapture is possible (CIR), and sometimes it is not (CDR). We will now consider both of these possibilities.

Capture, Immediate Re-capture (CIR)
We will consider three kinds of CIR:

1) Snapback, except those in (3), below.
2) In-line capture-immediate-recapture seen in hanezeki; and
3) A special example of snapback – the capture of a single stone in the corner to give one of the positions in simple corner nakade with eye – see Fig 19, which becomes Fig 20 -- there are only two kinds of these. They are examined in a paper on seki with nakade [Fearnley2005b]..

CIR—snapback
Snapback (uttegaeshi(J), hwangyeok(K), daotei/daopu(C)) is a well known example of a CIR position – mutual capture is possible, and one side can recapture immediately. There seems to be no systematic study of seki with snapback. Fig 21 is an in-line snapback -- these cannot help us construct seki. Fig 22 shows a diagonal snapback in the centre of the board – if Black captures first then White has one less liberty than if White plays first. If White plays first in Fig 23, White gets one eye (2 liberties), but if Black plays first the White gets two eyes with ko – this might be used in creating seki configurations. Fig 24 is a precursor of the snapback in Fig 25. If White captures first in Fig 24, they get a choice of 2 liberties or a one-point eye. If Black plays first, we get Fig 25, and eventually White has a choice of 3 liberties, or ko for 2 eyes. Precursor positions are also useful to us.
Corner snapbacks differ from all those in the centre of the board – for example, in Fig 26 ("TSM"), after White captures the four stones, the two white stones can be recaptured with two plays. There is another configuration -- related unilaterally to snapback, and to capture delayed recapture (CDR, which we discuss later) – see Fig 27 – if Black captures White, White throws in again, and will get a "TSM" snapback similar to Fig 26. However, if White plays first in Fig 27 then Black can still reduce White's eye space to one eye. [Fearnley2004] has done a preliminary analysis simply involving these CDR configurations, as well as (CIR) snapback, and their precursor positions – for example, Fig 23 shows a snapback which leads to a ko for two eyes. Various snapback positions can also be used to build sekis with capture.

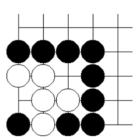


Fig 19: pre-1-eye

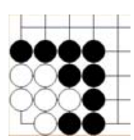


Fig 20: Flower-6

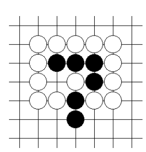


Fig 21: in-line

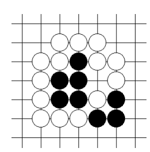


Fig 22: diagonal

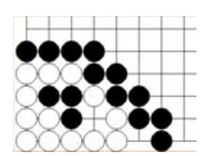


Fig 23: ko-for-2-eyes?

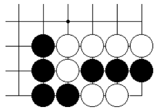


Fig 24: pre-snapback

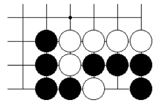


Fig 25: snapback

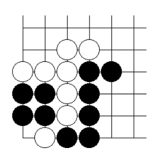


Fig 26: TSM

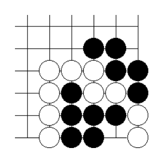


Fig 27: half-snapback

CIR – hanezeki
Hanezeki (jeochim bik (K)) has apparently been known for more than 650 years. Fig 28 appears in some editions of *XuanXuan QiJing* [Yan1347], and also in *Guan Zi Pu* [Guo1689]. Another previously published

example of hanezeki is shown in Fig 29 – from [Shimada1958]. Both are discussed in [Ikeda1992]. In both of these examples, if either player captures any stones, then all their stones will eventually be killed.
Igo Hatsuyoron [Dosetsu1713] contained another 3-stone hanezeki – see Fig 30. This problem was lost, rediscovered in 1982 by FUJISAWA Shuko, published by him in [Fujisawa1993], and also discussed in [GW1982], and [Feldmann2005]. It is a beautiful full-board problem, where White is eventually very unwilling to capture 20 black stones – to do so loses *all* the white stones in the bottom right quarter of the board.

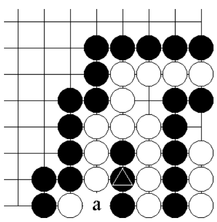


Fig 28: XuanXuan QiJing (1347)

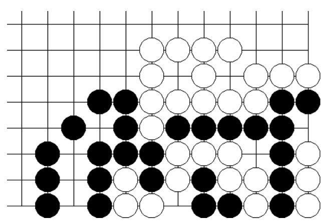


Fig 29: Shimada (1958)

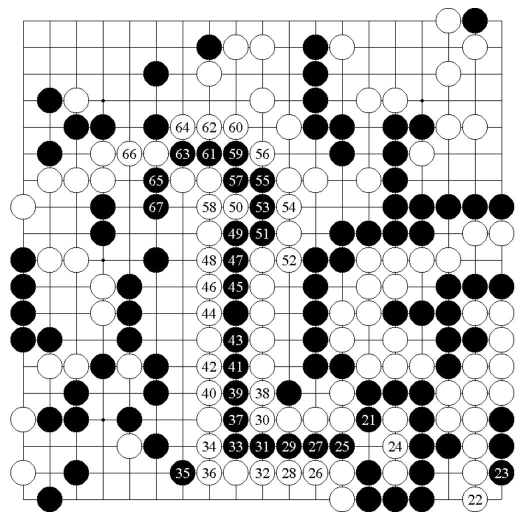


Fig 30: Igo Hatsuyoron (1713)

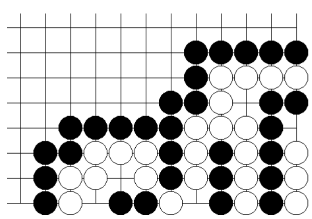


Fig 31a: Linked seki (Shimada)

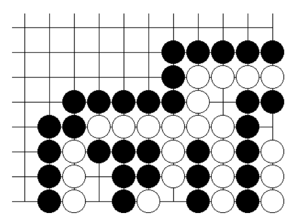


Fig 31b: another linked seki

We can generalize the shapes seen in Fig 28 and Fig 29. These consist of two components – an ordinary nakade component, and a special mutual-capture component. The mutual-capture component behaves like a nakade capture for one player, but for the other player, after they make a capture, their capturing stones can be re-captured immediately. If such a position is a seki then this capture-recapture must not be favourable for the other player – this may depend on the size/shape of the initial capture.
In [Fearnley2005a] there is a complete analysis of generalized hanezeki showing 130 different configurations (and a further 87 depending on ko) where such shapes are coupled with nakade to produce a seki – of these 20 (and a further 28 depending on ko) are in the corner only. Wrap-around configurations (see Fig 37, and discussion below) are not counted as being different. The nakade captures do not have to be of three stones – [Fearnley2005a] shows examples with captures of from three to six stones, though those with 5-, and 6-stone nakade are more unusual, and are also more likely to involve ko.

Fig 32: Hat-4 -- all shapes/sizes **Fig 33: Hat-4 -- other** **Fig 34: Smallest “largest” hanezeki?**

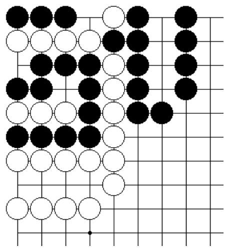
Fig 32 illustrates the example of a hat-4 nakade in the centre of the board, coupled with several distinct mutual captures. We can do the same with all the other nakade, of any size/shape. Note that “b1”, “b2”, and “b3” are equivalent to each other. “e/f” is meant to represent a capture which gives White one (“e”), or two (“f”), more liberties than the 5-stone capture in “d” – in reality these have to be bulkier than shown -- see Fig 34 for a candidate for the smallest possible “e” – the smallest “f” will have to be even more bulky (but maybe slightly smaller than in Fig 36). All of “a”-“c” can also exist at the edge, though “d”-“f” cannot – at the edge they may behave differently (Fig 39 is such a case). In general, the captures “a”-“f” may give different results from each other. Note that each of these is simply a representative of a larger class – for example, in Fig 30, if we are concerned primarily with the status of the 12-stone black group including move 21, then the 20-stone capture in Fig 30 is equivalent to any 2-, or 3-stone capture (compare “b1”-“b3” in Fig 32). With the particular 3-stone nakade in Fig 30, the black capture of a single stone, as in “a” (Fig 32), gives the same result as a 2-stone capture as in “b1” – both are hanezeki. Therefore White also cannot safely play 26 at 27 in Fig 30.

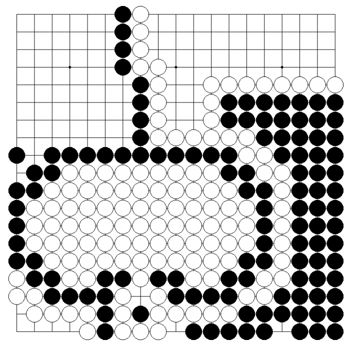
To make a hanezeki from the capture-recaptures in Fig 32, White’s nakade capture must be the same as Black’s nakade capture – hat-4, in this case.

Knowing the size, and shape, of the black nakade is not enough – its relationship to the white stones in the capture-recapture component is significant. Fig 32 illustrates some differences between the captures “a”-“f”. With this particular black nakade, in this particular orientation: “a”, and “b”, both result in hanezeki with (different) ko involvement; “c”-“f” are all simple hanezeki. Fig 33 shows another important difference: the configuration at the top gives Black an outright win – it is *not* hanezeki. However, if Black can capture more than one stone (i.e. “b”-“f”) then it is unsafe for Black – i.e. it is a normal hanezeki. The other two arrangements in Fig 33 behave exactly as the hat-4 in Fig 32, when we replace the single white stone in Fig 33 with each of b”-“f” in Fig 32.

With most of the capture/recapture components we can create a hanezeki when we capture one of the lumps “a”-“d”. However, we need “e” for one of the orientations of the hammer-5 nakade, and for all the ways of capturing a flower-six (hana roku(J)/ juliu(C)/ maehwayukgung(K)) nakade. See Fig 34 with a flower-6 nakade, coupled with a candidate “e” – maybe the white lump is the smallest one which guarantees the necessary extra liberty. The size of the black group is chosen to balance that of the white capture-recapture component – if the black group were any smaller, Black would not leave it as a seki, but would prefer to capture the white stones in the capture-recapture component. This may be the largest hanezeki where all the stones are necessary – i.e. the smallest of the largest hanezeki. This contrasts with the smallest possible hanezeki (Fig 35).

Fig 36 is another, even larger, configuration – an unstable seki, but not a proper hanezeki -- in which an “f”-type capture could give a better result than an “e”-type capture: Black has a one-point eye, and *White* can capture a single stone. The position is potentially seki, however -- White can safely play elsewhere, provided that the loss of a large lump of stones is less than Black's loss. It seems likely that the 75-stone lump of white stones could be replaced by one of about 50 stones.





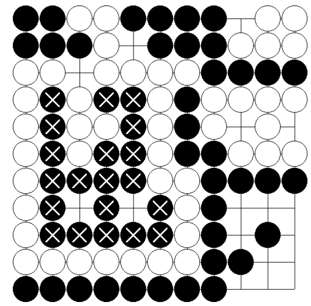
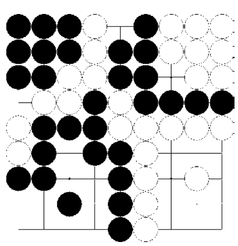


Fig 35: Smallest hanezeki **Fig 36: White captures one stone?** **Fig 37:** ***Wrap-around***

In Fig 37 the capture of four white stones is the same as in "c" in Fig 32. However, the only way to build such a capture for the corner configuration shown here is to have the white stones *wrap around* the black stones marked with "X". The wrap-around can also be used with all centre nakade configurations, and with some both on the edge, and in the corner. However, it is not required for any but a few in the corner, such as Fig 37.

[Fearnley2005a] finds the unstable complex corner hanezeki, shown in Fig 38, where White has a defence of double ko, but Black can profit from the initial capture. It is the only hanezeki with double-ko as a defence. There are 87 hanezeki where the defender needs (non-removable) ko threats to survive. The one which requires the most ignored ko threats is shown in Fig 39, where White needs at least 10 ko threats, and must ignore 9 moves by Black!



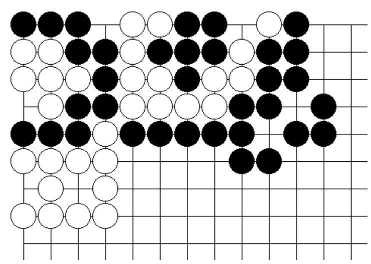
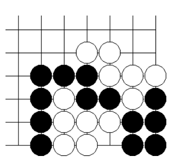


Fig 38: Double Murashima's ko **Fig 39: Most ko threats needed**

We do not know of any published analysis of other possible combinations – for example, mutual-capture hanezeki components opposing other hanezeki components, or opposing non-nakade captures.



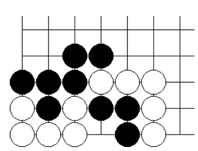


Fig 40: CDR **Fig 41: eye v 2-eyes-with-ko**

Capture, Delayed Re-capture (CDR)

There are several other examples of positions where mutual immediate capture is possible, but where (for both players) the capture alone does not guarantee independent life – because the capturing stones can be re-captured, either directly, or by reducing the eyes. We call these positions "Capture, Delayed Re-capture" (CDR). We do not know of any complete catalogue of such positions. In a study of CDR in seki, the first step is similar to that in CIR. We need to identify captures which do not *guarantee* life (either two eyes or a self-contained seki) – most outcomes are possible – e.g. ko for two eyes, one large eye, no eyes but several liberties, …, an adverse approach-move ko. Where a player has a choice between such outcomes, it will depend on the wider context. In the corner, especially with cutting points, forced sequences can lead to the creation of killable 7-nakade (or larger nakade) with two, or more stones inside. [Fearnley2004] has examined some such configurations where both players may capture the other immediately, but where there is only delayed re-capture (CDR) – two simple example are shown in Fig 40 and Fig 41. All of these CDR may be combined, either with others, or with immediate re-capture (CIR) configurations, to create positions of potential interest.

Conclusions, extensions, and further work

In seki, as in all fights between groups, players try to maximize *both* eyes and liberties (maybe with ko). These sub-goals may be in conflict – the choice depends on which satisfies the primary goal (life of group). [Landman1995] extends the application of Combinatorial Games Theory (CGT) to Go ([Berlekamp1994], [Müller1996]) to the study of eyes, including consideration of liberties, ko, and seki (with only two groups). He outlines what needs to be done to extend CGT techniques to the study of liberties, and larger sekis. It seems to us that a major remaining challenge is to integrate both eyes and liberties into a fully unified framework. We could then hope to calculate eye-liberty values separately for all components of a (potential) seki, so that we could then identify (un)safe moves where we have combinations of many groups.

Much more work needs to be done – after completing the work with immediate captures, a first step would be to extend the analysis to include situations where the first capture is not immediate. In doing this we would start with those configurations which could lead to those where the first capture happens immediately. Such an analysis would also have to incorporate consideration of non-symmetric liberties. It could be expected to include positions where *mutual* delayed capture is possible -- it is not yet clear that these can be part of shared life configurations. Perhaps work on delayed mutual capture should wait until there is a complete catalogue of all shapes that immediately follow a capture, and which do not give guaranteed life -- including those with cutting points, and those in the corner. Computerized searches may be needed to create complete catalogues/databases of dead, and (minimal) live positions.

It will eventually be necessary to include configurations which have *initial* loopy/cyclic positions, including double-ko seki

Hopefully, it will be possible to extend the work of [Gurvich1981] to find all simple fully-connected seki – in these, a capture implies that the capturing chain gets independent, unconditional, life. Perhaps, their ideas can be extended to include captures which do not guarantee independent life, although it seems likely that extending the CGT techniques of [Berlekamp1994], [Landman1995], and others, will be more fruitful.

Further work relating to this paper will appear at [Fearnley2005c].

References

Berlekamp1994: Elwyn R BERLEKAMP, and David WOLFE, *Mathematical Go Endgames*, Ishi Press; 1994. Also published as *Mathematical Go* by A.K.Peters, 1994

Bouzy2001: Bruno BOUZY, Tristan CAZENAVE, *Computer Go: an AI Oriented Survey,* Artificial Intelligence Journal 132(1): 39-103 (2001) – available online via http://www.math-info.univ-paris5.fr/~bouzy/publications.html

Bratko1980: Ivan BRATKO, and Donald MICHIE, *An advice program for a complex Chess programming task*, The Computer Journal, Vol 23, no 4, pp 353-359, 1980

Dosetsu1713: Inoue DOSETSU Inseki, *Igo Hatsuyoron* (also known as *Fudanzakura*(J)/ *FaYangLun*(C)/ *Bal Yang Hyun. Gam Non*(K)), 1713.

Fearnley2001: http://www.goban.demon.co.uk/go/bestiary/brain_hurts.html -- a web page largely devoted to reporting the much earlier work of Ger HUNGERINK, which was originally published in “Go”, the Journal of the Dutch Go Association.

Fearnley2003: Harry FEARNLEY, (Unpublished manuscript). *Catalogue of topologically distinct terminal arrangements of from one to six 2-eyed groups.*

Fearnley2004: Harry FEARNLEY, (Unpublished manuscript). *Preliminary notes on eyes and liberties in mutual capture, delayed re-capture.*

Fearnley2005a: Harry FEARNLEY. *Shared life in Go – generalized hanezeki*. : http://www.goban.demon.co.uk/go/seki/hanezeki/hanezeki.html
Fearnley2005b: Harry FEARNLEY. *Shared life in Go – generalized nakade*. http://www.goban.demon.co.uk/go/seki/nakade/nakade.html
Fearnley2005c: Harry FEARNLEY. *Shared life in Go – an overview*. http://www.goban.demon.co.uk/go/seki/overview/overview.html
Feldmann2005: Denis FELDMANN, *Commented bestiary* -- http://perso.wanadoo.fr/denis.feldmann/bestiary.htm#p5
Fujisawa1993: FUJISAWA Shuko, *Igo Hatsuyoron* (Japanese language), 2nd edition, published 1993. ISBN4-582-80412-8. First edition was 1982.
GoBase: GoBase : a searchable, online, database at http://gobase.org/
Grant1984: Andrew GRANT: *Go Paradoxes*, British Go Journal, No 62, p 23, see http://www.britgo.org/bgj/06223.html
Guo1689: GUO BuiLing*, Guan Zi Pu* (*Kanzufu*(J)/ *Gwan Ja Bo*(K) (Chinese language), 1689.
Gurvich1981: Vladimir GURVICH, and Andrey GOL'BERG, *Seki in Go*, transcription of a paper, and letter to Prof Klaus Heine, 1981 -- http://www.goban.demon.co.uk/go/seki/gurvich/g-g_cover.html
GoWorld1982: Go World No 29, 1982, pp 43, 47-49, publisher Ishi Press.
Hansen1989: *Japanese Rules of Go, 1989* – http://www-2.cs.cmu.edu/~wjh/go/rules/Japanese.html due to Fred HANSEN – see example 13
Haruyama1979: HARUYAMA Isamu (9 dan), *Strange and Wonderful Shapes*, in *Go World*, Winter 1987-88, No 50, pp 16-22, translated into English from the Japanese language magazine *Igo Club* (Japanese language), of January 1979.
Ikeda1992: IKEDA Toshio**,** *On The Rules of Go*, published 1992 by Fujitsu, both English and Japanese -- a collection of articles from September 1968 to November 1969, originally published in the Japanese language magazine *Igo Shincho* – http://gobase.org/studying/rules/ikeda/e_s2/e2030802.html (English)
Ing1991: ING Chang-ki, *The SST Laws of Wei-Ch'i*, translated by James DAVIES. 1991, Published by Ing Chang-Ki Wei Ch'i (Go) Educational Foundation, Taipei, Taiwan. Also available at http://www-2.cs.cmu.edu/~wjh/go/rules/SST.html.
Landman1995: Howard A. LANDMAN, *Eyespace values in Go*, Combinatorial Games, MSRI Publications, p203-233 – also http://www.polyamory.org/~howard/Go/Eyespace.ps. Also published in Games of no Chance, edited by Richard NOWAKOWSKI, publ MSRI 1994, and online at http://www.msri.org/publications/books/Book29/contents.html
Müller1996: Martin MŰLLER, Elwyn R BERLEKAMP, and Bill SPIGHT. *Generalized thermography: Algorithms, implementation, and application to Go endgames*. Technical Report 96-030, ICSI Berkeley, 1996. http://www.cs.ualberta.ca/~mmueller/publications.html#CITEMuelleretal1996
Murashima1973: MURASHIMA Yoshinori: *Ko Jiten* (Japanese lang), Kaibundo Shinkosha, Tokyo 1973
Pauli2004a: Robert PAULI, *Biggest Known Eye Space For Which There Is A Nakade,* http://senseis.xmp.net/?BiggestKnownEyeSpaceForWhichThereIsANakade
Pauli2004b: Robert PAULI, *Cubic Seki* http://senseis.xmp.net/?CubicSeki
RGG: rec.games.go newsgroup at news://rec.games.go or http://groups.google.co.uk/group/rec.games.go
SenseisLibrary: Sensei's Library: http://senseis.xmp.net -- a searchable online resource dedicated to Go.
Shimada1958: SHIMADA Takuji, *Igo no Suri* (Japanese language), revised edition, published by Maki Shoten 1958, pp 99-100 (first edition published by Kawade Shobo, 1943). A translation into English of Chapter 6 can be found at http://www.goban.demon.co.uk/go/shimada/chap6.html
Yan1347*:* YAN TianZhang, and YAN DeFu*, XuanXuan QiJing* (*Gengen Gokyo*(J)/ *Hyun Hyun Ki Kyung* (K)), (Chinese language), first published in 1347.